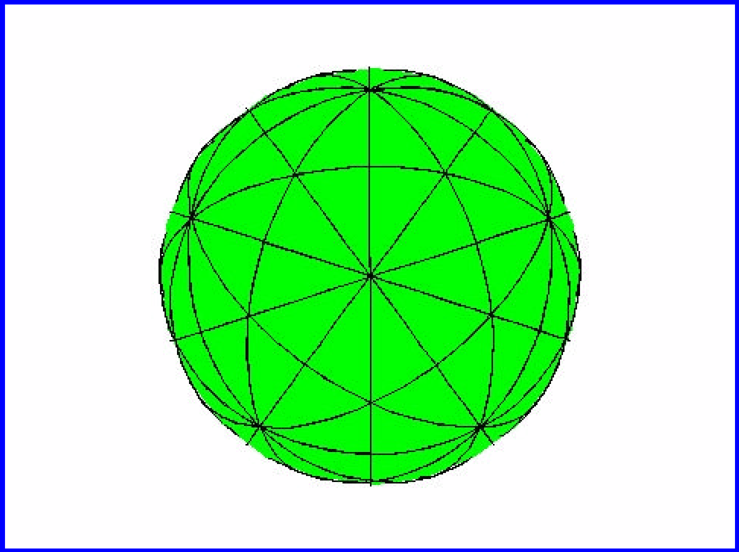

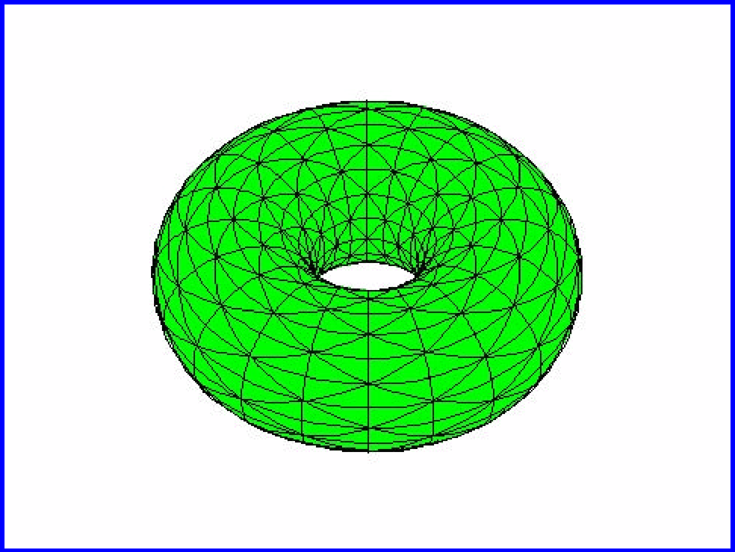

Higher genus Soccer Balls and Kaleidoscopic Tilings in the Hyperbolic Plane

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Outline

- **Talk 1** The relation between higher genus soccer balls and the kaleidoscopic tilings
- **Talk 2** Divisible tiling in the hyperbolic plane
- all of this work has been done jointly with undergraduates

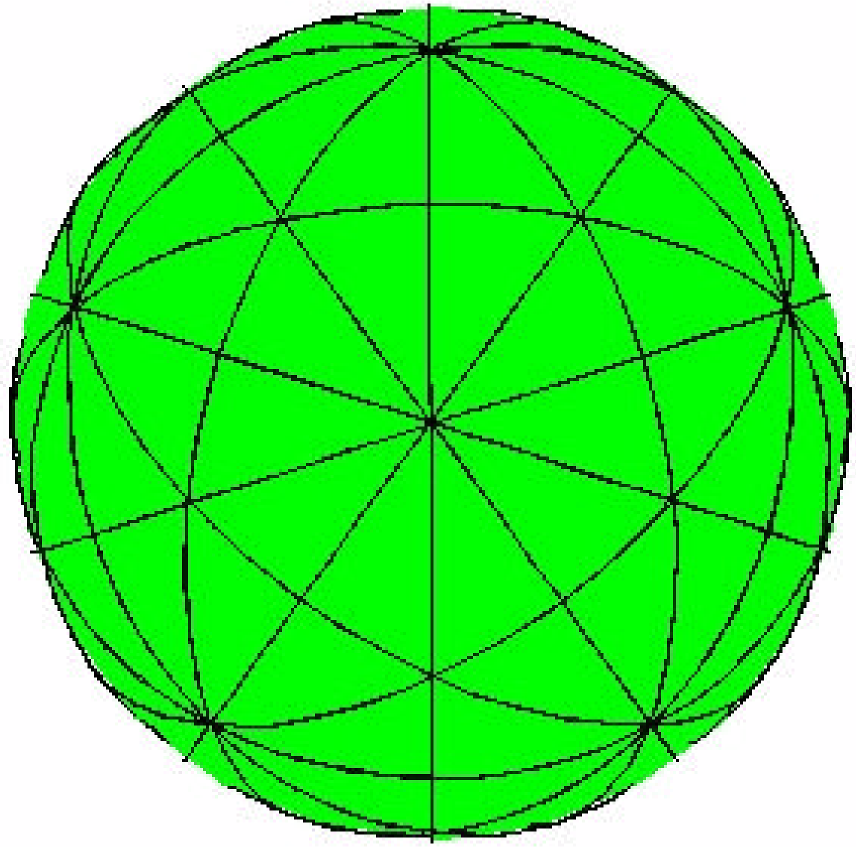
First Talk: Soccer Ball and Tilings

- Analyze soccer ball (and torus soccer ball)
- Riemann-Hurwitz equation and Euler characteristic
- tilings in the Euclidean and hyperbolic plane
- Making higher genus soccer balls
- geometric structure vs. algebraic structure
  - punch line group theory wins the day

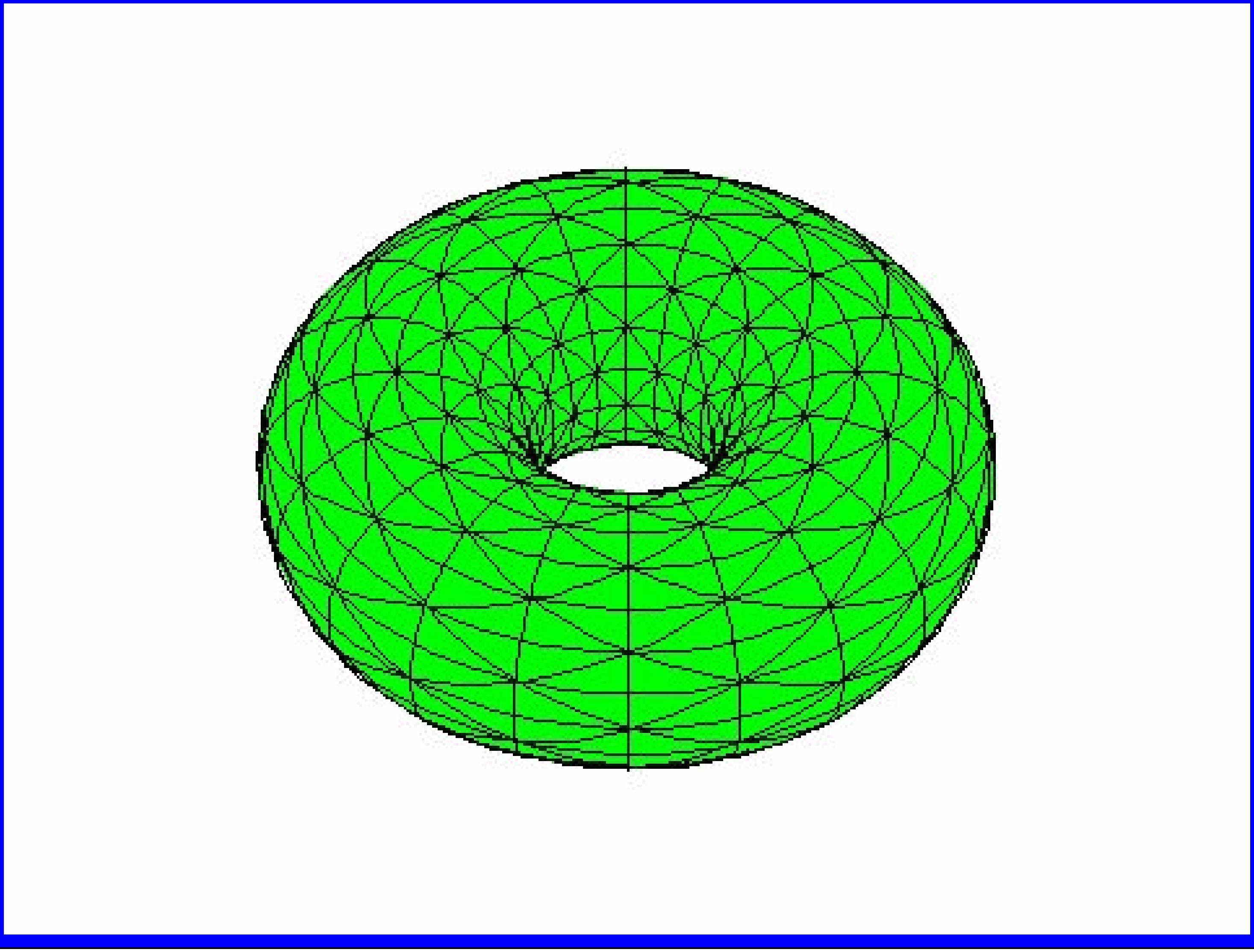
The soccer ball

- show ball
- show tiling of soccer ball
- show tiling of torus
- define tiling
- from tiling to soccer ball

Icosahedral-Dodecahedral Tiling (2,3,5) - tiling



(2,4,4) -tiling of the torus



Tiling: Definition

- Let **S** be a surface of genus ***s*** .
- **<u>Tiling:</u>** Covering by polygons "without gaps and overlaps"
- **<u>Kaleidoscopic:</u>** Symmetric via reflections in edges.
- **<u>Geodesic:</u>** Edges in tiles extend to geodesics in both directions
- terminology: (l,m,n) -triangle,

Tiling to soccer ball

- standard soccer ball example
- non-standard soccer ball
- torus example

Riemann Hurwitz equation (euler characteristic proof)

Let S be a surface of genus s and $2|G|$ the number of triangles:

$$\frac{2s-2}{|G|} = 1 - \frac{1}{l} - \frac{1}{m} - \frac{1}{n}$$

Hyperbolic triangles

- hyperbolic when $s \geq 2$ or

$$\frac{1}{l} + \frac{1}{m} + \frac{1}{n} < 0$$

Hyperbolic geometry

- Points, lines and angles
- reflections - show picture

Tilings of Euclidean and Hyperbolic Plane - Examples

- Euclidean
  - (2,3,6) example
  - (2,4,4) example
  - (3,3,3) example
- hyperbolic
  - (2,3,7) example
  - (3,3,4) example

Soccer Ball Patterns

- (2,3,6) patterns
- (2,4,4) patterns
- (2,3,7) pattern

Making Soccer Balls

- Show picture of making surface from a tiling
- words to label tiles
- word relations give a surface construction recipe

Word relations 1

Define:

$$a = pq,\ b = qr, c = rp$$

There are universal relations
and surface relations

Word relations 2
Universal Word Relations

$$p^2 = q^2 = r^2 = 1.$$

$$a^l = b^m = c^n = 1,$$

$$abc = 1, (pqqrrp = 1)$$

$$q(a) = qaq^{-1} = qpqq = qp = a^{-1},$$

$$q(b) = qbq^{-1} = qqrq = rq = b^{-1}.$$

Word relations 3
Additional relations for a surface

- Example Suppose we have $l=m=n=4$ and $ab=ba$ then $|G| = 16$.

- **Reason:** There is a map

$$\{\text{words in a,b,c}\} \dashrightarrow G = Z_4 \times Z_4$$

where Z_4 = integers mod 4.

Punch line

- Group theory exactly explains the structure of tiled surfaces and soccer balls
- The very rich theory of groups gives a lot of power in exploring the structure of tilings and soccer balls